

求解叶轮机粘性流场的 再生解析数值方法*

哈尔滨工业大学 张池平** 冯国泰 崔明根

【摘要】 应用再生核方法求解叶轮机粘性流场是一种新颖的解析数值方法,它可以解决计算中的存贮、计算速度和精度问题。本文给出了用再生核方法求解叶轮机粘性流场的思路和方法,以及如何在曲线坐标系中应用再生核方法,并给出了流函数积分方程的求解过程。最后对平面扩压管道进行了数值试验,结果与有关文献是吻合的。

关键词: 叶轮 粘性流动 再生 数值计算

分类号: TK472

叶轮机粘性流场求解方法大都用有限差分方法、有限元法和边界元方法。这3种方法在求解中都存在存贮、计算速度和精度问题。为了解决这一问题,根本上讲还应是寻找解析解,但在这方面进展极其缓慢。能否把解析解与数值解有机结合起来呢?陈景仁^[1]在这方面作了尝试,但由于这种解法复杂没有推广。崔明根等人提出并给出了利用再生核进行解析数值计算的途径^[2,3],这种方法是很有效的。这种方法的实质是在 W_2^1, W_2^2 空间中利用再生核找出方程的级数形式的精确解表达式,级数截断得到近似解,但这一截断在给定点精确满足方程,取的点数多解的精度高。这是一种通用性很强的求解方法,由于这种方法具有解析解与数值解的双重特点,又是在再生核空间利用再生核求解,因此这种方法称为再生解析数值方法。由于这种方法求解精度高、省内存、省机时、没有数值粘性、适应局部粘性效应,因此采用这种方法求解叶轮机粘性流场是十分有意义、有发展前途的。最后本文对平面扩压管道应用再生解析数值方法进行了数值计算。

1 基本概念和求解方法

设 $D = [a, b] \times [c, d] \times [e, f]$ 是 R^3 中任一长方体, Γ 表示边界。空间 $W_2^1(D)$ 定义为: $W_2^1(D) = \{u(x, y, z) \mid u|_{\Gamma} = 0, u(x, y, z) \in L^2(D) \text{ 且为实的绝对连续函数; } u_x, u_y, u_z \in L^2(D)\}$ 。 $W_2^1(D)$ 中规定内积为:

$$(u, v) = \iiint_D v dx dy dz + \iint_{\Gamma} u_x v_x + u_y v_y + u_z v_z dx dy dz$$

其中: $u_{x_1} = u_x, u_{x_2} = u_y, u_{x_3} = u_z; v_{x_1} = v_x, v_{x_2} = v_y, v_{x_3} = v_z$ 。空间 $W_2^2(D)$ 也可同样定义。

再生核定义: 设 H 是 Hilbert 函数空间, 其元素是某个抽象集合 B 上的实值或复值函数。

如果 $M \in B, R_M(M')$ 作为 M' 的函数是 H 中的元素, 而且对任何 $M \in B$ 及 $u \in H$ 则有: $u(M) = (R_M(M'), u(M'))$ 。则称 $R_M(M')$ 是 Hilbert 函数空间 H 的再生核。

对于空间 $W_2^1(D), W_2^2(D)$ 的算子方程:

$$A u = f \quad (1)$$

据投影定理, u 的数值解 u_n 可以表示为 $u_n = \sum_{i=1}^n (u, \Psi_i) \Psi_i$ 。其中 $\Psi_i = \sum_{k=1}^i \beta_k \Psi_k, \Psi_k = A^* \Phi_k, A^*$ 是 A 的共轭算子, $\Phi_k = R_{M_k}(M)$ 。那么:

$$u_n = \sum_{i=1}^n \left[u, \sum_{k=1}^i \beta_k A^* \Phi_k \right] \Psi_i = \sum_{i=1}^n \sum_{k=1}^i \sum_{p=1}^i \beta_k \beta_p f_i \Psi_i \quad (2)$$

以上给出了 u 的数值解的求解公式。

对平面不可压粘性流, 在 $\mu = \text{const}$ 条件下平均方程及对应积分方程为:

$$\nabla^2 \Psi = -\Omega, \quad \partial \Omega / \partial n + \vec{V} \cdot \nabla \Omega = \vec{V} \cdot \nabla^2 \Omega \quad (3)$$

$$\iint_A \frac{\partial \Omega}{\partial n} dA + \oint \left[v_n \Omega - \vec{V} \cdot \frac{\partial \Omega}{\partial n} \right] dl = 0, \quad \oint \frac{\partial \Psi}{\partial n} dl = - \iint_A \Omega dA \quad (4)$$

(3) 式适用于 W_2^2 空间, 而 (4) 式适用于 W_2^1 空间。

由于利用再生核求解时边界条件为齐次边界条件, 对 (3)、(4) 式可有如下边界函数:

$$\Omega = \Omega_A(x, y), \quad \Psi = \Psi_A(x, y) \quad (\text{在 } \Gamma) \quad (5)$$

将 (5) 式代入 (3)、(4) 式, 并令 $\Omega = \Omega_A + \Omega_B, \Psi = \Psi_A + \Psi_B$ 。这样有方程 (在构造边界函数时取法向导数为零):

$$\nabla^2 \Psi = -\nabla^2 \Psi_A - \Omega_B - \Omega_C \quad (6)$$

$$(\partial \Omega / \partial n) + \vec{V} \cdot \nabla \Omega = \vec{V} \cdot \nabla^2 \Omega_A + \vec{V} \cdot \nabla^2 \Omega_B - \vec{V} \cdot \nabla \Omega_C \quad (7)$$

$$\iint_A \frac{\partial \Omega}{\partial n} dA + \oint \left[v_n \Omega - \vec{V} \cdot \frac{\partial \Omega}{\partial n} \right] dl = - \oint v_n \Omega_C dl \quad (8)$$

$$\oint \frac{\partial \Psi}{\partial n} dl = - \iint_A \Omega_B dA - \iint_A \Omega_C dA \quad (9)$$

另外, 再生核在推导和求解中是利用规整的矩形或长方体, 其适用的空间 W_2^1, W_2^2 等的内积也是在这种规整域上计算的, 因此在求解叶轮机复杂域时必须采用曲线坐标系下的方程, 其对应的计算域是规整的。

从以上可以看出, 在求解过程中关键是求解函数 $\Psi_i, i = 1, 2, \dots, \infty$ 。仅就方程 (9) 为例说明 Ψ_i 求解的方法。

为简单起见给出矩形域笛卡尔坐标系的 Ψ_i 求解。这时有:

$$\Psi_i = \left[\oint \frac{\partial R_{M'}(M)}{\partial n} dl, \Phi(M') \right] \quad (10)$$

第一步应给出 $R_{M'}(M)$ 和 $\Phi(M')$

$$R_{M'}(M) = a_1 e^{\xi_+ \eta} + a_2 e^{-\xi_+ \eta} + a_3 e^{\xi_- \eta} + a_4 e^{-\xi_- \eta} \quad (11)$$

$a_1 = (A/\Delta) e^{-2a}, a_2 = (B/\Delta) e^{-2c}, a_3 = -(A/\Delta), a_4 = -(B/\Delta), A = e^{x-2b} - e^{-x}, B = e^{x-2d} - e^{-y}, \Delta = 1/4(e^{-2b} - e^{-2a})(e^{-2d} - e^{-2c})$ 。 a, b 为求解域 x 方向上、下限, c, d 为求解域 y 方向上、下限。

$$\Phi = (M') = a_1 e^{\xi_+ \eta} + a_2 e^{\xi_- \eta} + a_3 e^{-\xi_+ \eta} + a_4 e^{-\xi_- \eta} \quad (12)$$

第二步求:

$$\oint \frac{\partial R_{M'}(M)}{\partial n} dl = D_1 e^{\eta} + D_2 e^{-\eta} + D_3 e^{\xi} + D_4 e^{-\xi} \quad (13)$$

第三步求 ψ : 由(12)、(13)作内积(即根据(10))得

$$\psi_i = Q_{1i}e^{x+y} + Q_{2i}e^{x-y} + Q_{3i}e^{-x+y} + Q_{4i}e^{-x-y} \quad (14)$$

知 ψ 后, 由(2)式就可求出流函数的解析数值解。

2 计算结果和分析

本文对平面扩压管道, 应用该解析数值方法进行了数值试验。扩压管道如图 1, 算例是文

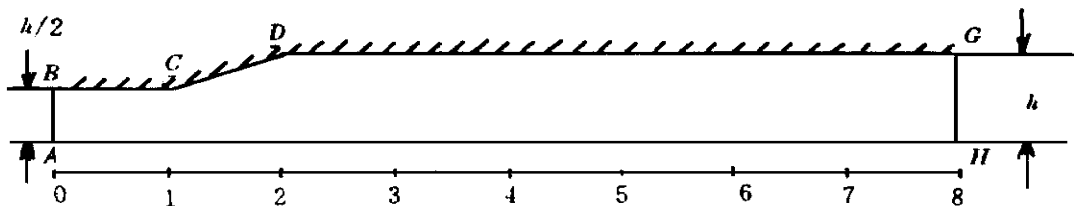


图 1 平面对称扩压管

献[4]给的结果。计算了雷诺数 $Re=60.0$ 和 $Re=6.0$ 中心线 AH 的速度分布如图 2, 和流函数分布如图 3。从图中可以看出应用这种解析数值方法计算的结果与文献[1]给出的结果吻合

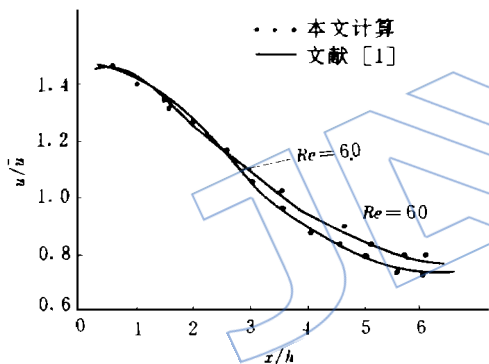


图 2 中心线 AH 的速度分布

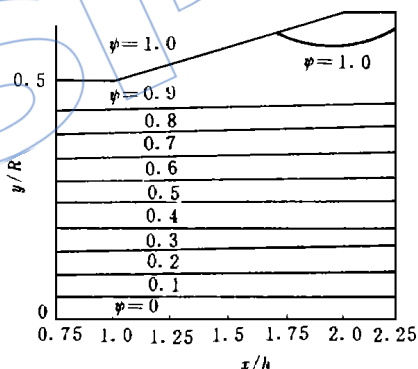


图 3 中心线 AH 的流函数分布

得很好。并且本文给出的方法在计算中, 解是显示计算, 不需要解方程组, 这就减少了计算过程中的存贮量, 避免了在数值计算中数值粘性的影响, 加快了计算速度。因此, 此解析数值方法用于粘性流场计算是有实用意义的。

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the foreign object, although the total pressure coefficient of this type intake slightly lessens. This type intake has extensive application in the future to the civil aviation with the strict requirements of airworthiness.

Key words Air Intake Gasdynamic performance Performance tests Bypass

A RESTORING ANALYTIC NUMERICAL METHOD FOR SOLUTION OF VISCOUS FLOW FIELD IN TURBINE

Zhang Chiping, Feng Guotai, Cui Minggen

(Dept. of Mathematics, Harbin University of Technology, Harbin 150001)

ABSTRACT

A new restoring analytic numerical method based on application of the method of restoring nucleus is developed for solution of viscous flow field in turbine. It makes the problems of storage, speed and accuracy in calculation easy to be solved. This paper offers ideas and methodology of application the method of restoring nucleus to solving viscous flow field in turbine, as well as in curvilinear coordinate system, and demonstrates the procedure of solving flow function integral equations. Finally, a numerical experiment for the two-dimensional cascade viscous flow mentioned in reference 1 by the aid of this method has been performed and the results are consistent with the data in reference. 1.

Key words Turbine Viscous flow Restoring Numerical computation

NUMERICAL SIMULATION OF 3- D FIELD WITHIN A COMPRESSOR ROTOR WITH C- H GRID

Wang Licheng

(2nd Dept., Nanjing University of Aeronautics and Astronautics, Nanjing 210016)

ABSTRACT

A 3-D C-H grid generation technique and 3-D Euler governing equation are described briefly. The highly loaded transonic compressor rotor designed by ONERA is taken as a numerical example. ONERA's numerical results and ours with H grid and C-H grid are compared respectively in respect of the pressure distributions along profiles at hub and tip of the rotor. The numerical technique presented is applied to evaluating the influence of inlet pre-swirl on its downstream flow field in the rotor, in result a new numerical approach is obtained for design optimization of pre-swirl distribution at compressor design stage. As numerical examples, two kinds of pre-swirl distributions are examined to show its influence on M_{w1} , β and α etc. distributions along radius.

Key words Euler's equation Three-dimensional flow Transonic compressor
Numerical simulation

EFFECT OF INTERBLADE PHASE ANGLE ON BLADE FLUTTER OF STEAM TURBINE

Zhang Yangjun, Li Kejian, Tao Deping

(4th Dept. Beijing University of Aeronautics and Astronautics, Beijing 100083)

ABSTRACT

The interblade phase angle is one of key factors to act on the aeroelastic stability of steam turbine. A two-dimensional numerical deforming actuator disk method on non-constant interblade phase angle along circumferential direction has been developed to predict the stall flutter of the steam turbine. The effect of the interblade phase angle on the blade flutter of the steam turbine has been studied with the help of the developed method. The results indicate that when the interblade phase angle is between -45° and $+90^\circ$, the stall flutter may occur at a certain